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U(2) Flavor Physics without U(2) Symmetry

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Abstract

We present a model of fermion masses based on a minimal, non-Abelian discrete symmetry that reproduces the Yukawa matrices usually associated with U(2) theories of flavor. Mass and mixing angle relations that follow from the simple form of the quark and charged lepton Yukawa textures are therefore common to both theories. We show that the differing representation structure of our horizontal symmetry allows for new solutions to the solar and atmospheric neutrino problems that do not involve modification of the original charged fermion Yukawa textures, or the introduction of sterile neutrinos.

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I. INTRODUCTION

One path toward understanding the observed hierarchy of fermion masses and mixing angles is to assert that at some high energy scale all Yukawa couplings, except that of the top quark, are forbidden by a new symmetry G_f that acts horizontally across the three standard model generations. As this symmetry is spontaneously broken to smaller subgroups at successively lower energy scales, a hierarchy of Yukawa couplings can be generated. The light fermion Yukawa couplings originate from higher-dimension operators involving the standard model matter fields and a set of ‘flavon’ fields ϕ , which are responsible for spontaneously breaking G_f . The higher-dimension operators are suppressed by a flavor scale M_f , which is the ultraviolet cut-off of the effective theory; ratios of flavon vacuum expectation values (vevs) to the flavor scale, $\langle\phi\rangle/M_f$, provide a set of small symmetry-breaking parameters that may be included systematically in the low-energy effective theory. Many models of this type have been proposed, with G_f either gauged or global, continuous or discrete, Abelian or non-Abelian, or some appropriate combination thereof [1]. Non-Abelian symmetries are particularly interesting in the context of supersymmetric theories, where flavor-changing neutral current (FCNC) processes mediated by superparticle exchange can be phenomenologically unacceptable [2]. If the three generations of any given standard model matter field are placed in $\mathbf{2}\oplus\mathbf{1}$ representations of some non-Abelian horizontal symmetry group, it is possible to achieve an exact degeneracy between superparticles of the first two generations when G_f is unbroken. In the low-energy theory, this degeneracy is lifted by the same small symmetry-breaking parameters that determine the light fermion Yukawa couplings, so that FCNC effects remain adequately suppressed, even with superparticle masses less than a TeV.

A particularly elegant model of this type considered in the literature assumes the continuous, global symmetry $G_f = U(2)$ [3–5]. Quarks and leptons are assigned to $\mathbf{2}\oplus\mathbf{1}$ representations, so that in tensor notation, one may represent the three generations of any matter field by $F^a + F^3$, where a is a $U(2)$ index, and F is Q , U , D , L , or E . A set of flavons is introduced consisting of ϕ_a , S_{ab} , and A_{ab} , where ϕ is a $U(2)$ doublet, and S (A) is a symmetric (antisymmetric) $U(2)$ triplet (singlet). If one assumes the pattern of vevs

$$\frac{\langle\phi\rangle}{M_f} = \begin{pmatrix} 0 \\ \epsilon \end{pmatrix}, \quad \frac{\langle S \rangle}{M_f} = \begin{pmatrix} 0 & 0 \\ 0 & \epsilon \end{pmatrix}, \quad \text{and} \quad \frac{\langle A \rangle}{M_f} = \begin{pmatrix} 0 & \epsilon' \\ -\epsilon' & 0 \end{pmatrix}, \quad (1.1)$$

which follows from the sequential breaking

$$U(2) \xrightarrow{\epsilon} U(1) \xrightarrow{\epsilon'} \text{nothing}, \quad (1.2)$$

then all fermion masses and Cabibbo-Kobayashi-Maskawa (CKM) mixing angles can be reproduced. More specifically, the pattern of vevs in Eq. (1.1) yields a Yukawa texture for the down quarks of the form

$$Y_D \sim \begin{pmatrix} 0 & d_1\epsilon' & 0 \\ -d_1\epsilon' & d_2\epsilon & d_3\epsilon \\ 0 & d_4\epsilon & 1 \end{pmatrix}, \quad (1.3)$$

where $\epsilon \approx 0.02$, $\epsilon' \approx 0.004$, and d_1, \dots, d_4 are $O(1)$ coefficients that can be determined from Ref. [5]. Differences between hierarchies in Y_D and Y_U can be obtained by embedding the

model in a grand unified theory [5]. For example, in an SU(5) GUT, one obtains differing powers of ϵ and ϵ' in the up quark Yukawa matrix by assuming that S_{ab} transforms as a **75**; combined GUT and flavor symmetries prevent A_{ab} and S_{ab} from coupling to the up and charm quark fields, unless an additional flavor singlet field Σ is introduced that transforms as an SU(5) adjoint. With $\langle \Sigma \rangle / M_f \sim \epsilon$, it is possible to explain why $m_d :: m_s :: m_b = \lambda^4 :: \lambda^2 :: 1$, while $m_u :: m_c :: m_t = \lambda^8 :: \lambda^4 :: 1$, where $\lambda = 0.22$ is the Cabibbo angle. The ratio m_t/m_b is assumed to be unrelated to U(2) symmetry breaking, and is put into the low-energy theory by hand.

In this letter we show that the properties of the U(2) model leading to the successful Yukawa textures described above are also properties of smaller discrete symmetry groups. To reproduce *all* of the phenomenological successes of the U(2) model, we require a candidate discrete symmetry group to have the following properties:

- **1, 2, and 3** dimensional representations.
- The multiplication rule **$2 \otimes 2 = 3 \oplus 1$** .
- A subgroup H_f such that the breaking pattern $G_f \rightarrow H_f \rightarrow \text{nothing}$ reproduces the canonical U(2) texture given in Eq. (1.3). This implies that an unbroken H_f -symmetry forbids all Yukawa entries with $O(\epsilon')$ vevs, but not those with $O(\epsilon)$ vevs.

In the next section we show that the smallest group satisfying these conditions is a product of the double tetrahedral group T' and an additional Z_3 factor. Since U(2) is isomorphic to $SU(2) \times U(1)$, it is not surprising that our candidate symmetry involves the product of a discrete subgroup of SU(2), T' , and a discrete subgroup of U(1), Z_3 . At this point, the reader who is unfamiliar with discrete group theory may feel somewhat uneasy.¹ We stress that the group T' is in fact a very simple discrete symmetry, a spinorial generalization of the symmetry of a regular tetrahedron (see Section II). It is worth noting that the charge assignments in the model we present render T' a nonanomalous discrete gauge symmetry, while the Z_3 factor is anomalous. Models based on non-Abelian discrete gauge symmetries have yielded viable theories of fermion masses, as have models based on discrete subgroups of anomalous U(1) gauge symmetries [1]. In the latter case it is generally assumed that the U(1) anomalies are cancelled by the Green-Schwarz mechanism in string theory [6]. It is interesting that our model turns out to be a hybrid of these two ideas.

One of the virtues of the model discussed in this letter is that it allows for elegant extensions that explain the solar and atmospheric neutrino deficits, while maintaining the original quark and charged lepton Yukawa textures. This distinguishes our model from the modified version of the U(2) model presented in Ref. [7]. Preserving the U(2) charged fermion textures is desirable since they lead to successful mass and mixing angle relations such as $|V_{ub}/V_{cb}| = \sqrt{m_u/m_c}$, which are ‘exact’ in the sense that they contain no unknown $O(1)$ multiplicative factors. Since we succeed in explaining solar and atmospheric neutrino oscillations without sacrificing the predictivity of the original model, we need not introduce sterile neutrinos, as in Ref. [8]. However, we do not try to explain simultaneously the more

¹For a review of basic terms, see Ref. [11].

controversial LSND results [9] in this paper. We will consider versions of our model that include sterile neutrinos in a longer publication [10].

II. THE SYMMETRY

We seek a non-Abelian candidate group G_f that provides the $\mathbf{2} \oplus \mathbf{1}$ representation (rep) structure for the matter fields described in the previous section. In order for the breaking of G_f to reproduce the U(2) charged fermion Yukawa texture in Eq. (1.3), one must have flavons that perform the same roles as ϕ_a , S_{ab} , and A_{ab} in the U(2) model. Since these are doublet, triplet, and nontrivial singlet reps, respectively, we require G_f to have reps of the same dimensions. Nontrivial singlets appear in all discrete groups of order < 32 [12], so we seek groups G_f with doublet and triplet representations.

The order 12 tetrahedral group T , the group of proper symmetries of a regular tetrahedron (which is also the alternating group A_4 , consisting of even permutations of four objects), is the smallest containing a triplet rep, but has no doublet reps. A number of groups with orders < 24 possess either doublet or triplet reps, but not both (See, for example, [12]).

It turns out that two groups of order 24 possess both doublet and triplet reps. One is the symmetric group S_4 of permutations on four objects, which is isomorphic to the group O of proper symmetries of a cube as well as the group T_d of all proper and improper symmetries of a regular tetrahedron. S_4 possesses two triplets $\mathbf{3}^\pm$, two singlets $\mathbf{1}^\pm$, and one doublet $\mathbf{2}$. However, in this case one encounters another difficulty: The combination rule for doublets in S_4 is $\mathbf{2} \otimes \mathbf{2} = \mathbf{2} \oplus \mathbf{1}^- \oplus \mathbf{1}^+$, which implies that the triplet flavon cannot connect two doublet fields such as those of the first two generations of Q and U . Thus, S_4 is not suitable for our purposes.

The unique group of order < 32 with the combination rule $\mathbf{2} \otimes \mathbf{2} \supset \mathbf{3}$ is the double tetrahedral group T' , which is order 24. The character table, from which one may readily generate explicit representation matrices, is presented in Table I. Geometrically, T' is the group of symmetries of a regular tetrahedron under proper rotations (Fig. 1). These symmetries consist of 1) rotations by $2\pi/3$ about an axis connecting a vertex and the opposite face (C_3), 2) rotations by π about an axis connecting the midpoints of two non-intersecting edges (C_2), and 3) the rotation R by 2π about any axis, which produces a factor -1 in the even-dimensional reps, exactly as in SU(2). Indeed, this feature is a consequence of $T' \subset \text{SU}(2)$, and the rotations C_3 and C_2 are actually of orders 6 and 4, respectively. Also, T' is isomorphic to the group $\text{SL}_2(\text{F}_3)$, which consists of 2×2 unimodular matrices whose elements are added and multiplied as integers modulo 3.

T' has three singlets $\mathbf{1}^0$ and $\mathbf{1}^\pm$, three doublets, $\mathbf{2}^0$ and $\mathbf{2}^\pm$, and one triplet, $\mathbf{3}$. The *triality* superscript describes in a concise way the rules for combining these reps: With the identification of \pm as ± 1 , the trialities add under addition modulo 3. In addition, the following rules hold:

$$\begin{aligned} \mathbf{1} \otimes \mathbf{R} &= \mathbf{R} \otimes \mathbf{1} = \mathbf{R} \quad \text{for any rep } \mathbf{R}, & \mathbf{2} \otimes \mathbf{2} &= \mathbf{3} \oplus \mathbf{1}, \\ \mathbf{2} \otimes \mathbf{3} &= \mathbf{3} \otimes \mathbf{2} = \mathbf{2}^0 \oplus \mathbf{2}^+ \oplus \mathbf{2}^-, & \mathbf{3} \otimes \mathbf{3} &= \mathbf{3} \oplus \mathbf{3} \oplus \mathbf{1}^0 \oplus \mathbf{1}^+ \oplus \mathbf{1}^-. \end{aligned} \quad (2.1)$$

Trialities flip sign under Hermitian conjugation. Thus, for example, $\mathbf{2}^+ \otimes \mathbf{2}^- = \mathbf{3} \oplus \mathbf{1}^0$, and $(\mathbf{2}^+)^{\dagger} \otimes \mathbf{2}^- = \mathbf{3} \oplus \mathbf{1}^+$.

One must now determine whether it is possible to place a sequence of vevs hierarchically in the desired elements of the Yukawa matrices. Notice if G_f is broken to a subgroup H_f that rotates the first generation matter fields by a common nontrivial phase, then H_f symmetry forbids all entries with $O(\epsilon')$ vevs in Eq. (1.3). Therefore, we require that the elements of G_f defining this subgroup have two-dimensional rep matrices of the form $\text{diag}\{\rho, 1\}$, with $\rho = \exp(2\pi i n/N)$ for some N that divides the order of G_f and some integer n relatively prime with respect to N . This form for ρ follows because reps of finite groups may be chosen unitary, and must give the identity when raised to the power of the order of G_f . Such elements generate a subgroup $H_f = Z_N$ of G_f . Whether such elements exist in G_f can be determined since the rep of any element can be brought to diagonal form by a basis transformation, while the eigenvalues $\rho, 1$ are invariant under such basis changes.

Even if a given element $C \in G_f$ has the diagonal form $\text{diag}\{\rho_1, \rho_2\}$, $\rho_i = \exp(2\pi i n_i/N)$ (and thus generates a subgroup, Z_N^C , of G_f), a phase rotation of the form $\text{diag}\{\rho, 1\}$ can be achieved if the original G_f is extended by forming a direct product with an additional factor Z_N . We then identify H_f as a subgroup of $Z_N^C \times Z_N$. We choose one element of the additional Z_N to compensate the phase of the 22 element of C , and similarly for the other elements of the Z_N^C . The element corresponding to C in $G_f \times Z_N$ then effectively acts upon the doublet as $\text{diag}\{\exp[2\pi i(n_1 - n_2)/N], 1\}$, and the remaining symmetry is $Z_{N/\text{gcf}(N, |n_1 - n_2|)}$. In the case that $|n_1 - n_2|$ and N are relatively prime, this reduction amounts to forming the diagonal subgroup Z_N^D of $Z_N^C \times Z_N$. Similar arguments apply to the singlet and triplet reps.

In the particular case of $G_f = T'$, one finds elements C that generate either Z_2 or Z_3 subgroups. By introducing an additional Z_n (with $n = 2$ or 3) one can arrange for a Z_n subgroup that affects only the first generation fields. In the case of Z_2 , the nontrivial element of the diagonal subgroup is of the form $\text{diag}\{-1, 1\}$, which leaves the 11 and 22 entries of the Yukawa matrices invariant. The incorrect relation $m_u = m_c$ then follows. On the other hand, Z_3 prevents an invariant 11 entry, so we are led to adopt

$$G_f = T' \times Z_3. \quad (2.2)$$

The reps of G_f are named by extending the notation for T' to include a superscript indicating the Z_3 rep. These are the trivial rep 0, which takes all elements to the identity, and two complex-conjugate reps $+$ and $-$. Like the trialities, these indices combine via addition modulo 3. We adopt the convention that the $T' \times Z_3$ reps $\mathbf{1}^{00}$, $\mathbf{1}^{+-}$, $\mathbf{1}^{-+}$, $\mathbf{2}^{0-}$, $\mathbf{2}^{++}$ and $\mathbf{2}^{-0}$ are special, in that these singlet reps and the second component of the doublets remain invariant under Z_3^D . Thus any $\mathbf{2} \oplus \mathbf{1}$ combination of these reps is potentially interesting for model building.

III. A MINIMAL MODEL

The minimal model has the three generations of matter fields transforming as $\mathbf{2}^{0-} \oplus \mathbf{1}^{00}$ under $G_f = T' \times Z_3$. The Higgs fields $H_{U,D}$ are pure singlets of G_f and transform as $\mathbf{1}^{00}$. Given these assignments, it is easy to obtain the transformation properties of the Yukawa matrices,

$$Y_{U,D,L} \sim \left(\frac{[\mathbf{3}^- \oplus \mathbf{1}^{0-}]}{[\mathbf{2}^{0+}]} \middle| \frac{[\mathbf{2}^{0+}]}{[\mathbf{1}^{00}]} \right). \quad (3.1)$$

Eq. (3.1) indicates the reps of the flavon fields needed to construct the fermion mass matrices. They are $\mathbf{1}^{0-}$, $\mathbf{2}^{0+}$, and $\mathbf{3}^-$, which we call A , ϕ , and S , respectively. Once these flavons acquire vevs, the flavor group is broken. We are interested in a two-step breaking controlled by two small parameters ϵ , and ϵ' , where

$$T' \otimes Z_3 \xrightarrow{\epsilon} Z_3^D \xrightarrow{\epsilon'} \text{nothing} . \quad (3.2)$$

Since we have chosen a doublet rep for the first two generations that transforms as $\text{diag}\{\rho, 1\}$ under Z_3^D , only the 22, 23, and 32 entries of the Yukawa matrices may develop vevs of $O(\epsilon)$, which we assume originate from vevs in S and ϕ . The symmetry Z_3^D is then broken by a $\mathbf{1}^{0-}$ vev of $O(\epsilon')$. The Clebsch-Gordan coefficients that couple a $\mathbf{1}^{0-}$ to two $\mathbf{2}^{0-}$ doublets is proportional to σ_2 , so the ϵ' appears in an antisymmetric matrix. We therefore produce the U(2) texture of Eq. (1.3). Since the $\mathbf{1}^{0-}$ and $\mathbf{3}^-$ flavon vevs appear as antisymmetric and symmetric matrices, respectively, all features of the grand unified extension of the U(2) model apply here, assuming the same GUT transformation properties are assigned to ϕ , S , and A . One can also show readily that the squark and slepton mass squared matrices are the same as in the U(2) model.

It is worth noting that we could construct completely equivalent theories had we chosen to place the matter fields in reps like $\mathbf{2}^{++} \oplus \mathbf{1}^{00}$ or $\mathbf{2}^{-0} \oplus \mathbf{1}^{00}$, which have the same transformation properties under Z_3^D as our original choice. The reps $\mathbf{2}^{0-} \oplus \mathbf{1}^{00}$ are desirable in that they fill the complete SU(2) representations $\mathbf{2} \oplus \mathbf{1}$, if we were to embed T' in SU(2). Since anomaly diagrams linear in this SU(2) vanish (and hence the linear Ibáñez-Ross condition is satisfied [13]), we conclude that T' is a consistent discrete gauge symmetry [14]. The additional Z_3 may also be considered a discrete gauge symmetry, providing its anomalies are cancelled by the Green-Schwarz mechanism.

IV. NEUTRINOS

In this section, we show that the model presented in Section III can be extended to describe the observed deficit in solar and atmospheric neutrinos. We consider two cases:

Case I: Here we do not assume grand unification, so that all flavons are SU(5) singlets. This case is of interest, for example, if one is only concerned with explaining flavor physics of the lepton sector. We choose

$$\nu_R \sim \mathbf{2}^{0-} \oplus \mathbf{1}^{-+}. \quad (4.1)$$

Note that the only difference from the other matter fields is the representation choice for the third generation field. The neutrino Dirac and Majorana mass matrices then have different textures from the charged fermion mass matrices. Their transformation properties are given by

$$M_{LR} \sim \left(\begin{array}{c|c} [\mathbf{3}^- \oplus \mathbf{1}^{0-}] & [\mathbf{2}^{+0}] \\ \hline [\mathbf{2}^{0+}] & [\mathbf{1}^{-+}] \end{array} \right) , \quad M_{RR} \sim \left(\begin{array}{c|c} [\mathbf{3}^-] & [\mathbf{2}^{+0}] \\ \hline [\mathbf{2}^{+0}] & [\mathbf{1}^{-+}] \end{array} \right) . \quad (4.2)$$

Note that we obtain the same triplet and nontrivial singlet in the upper 2×2 block as in the charged fermion mass matrices, as well as one of the same flavon doublets, the $\mathbf{2}^{0+}$; the

rep $\mathbf{1}^{0-}$ is not present in M_{RR} , since Majorana mass matrices are symmetric. In addition we obtain the reps $\mathbf{2}^{+0}$, $\mathbf{1}^{+-}$, and $\mathbf{1}^{-+}$, which did not appear in Eq. (3.1). New flavon fields can now be introduced with these transformation properties, and their effects on the neutrino physics can be explored. Let us consider introducing a single² new flavon ϕ_ν transforming as a $\mathbf{2}^{+0}$ and with a vev

$$\langle \phi_\nu \rangle \sim \sigma_2 \begin{pmatrix} \epsilon' \\ \epsilon \end{pmatrix}, \quad (4.3)$$

where σ_2 is the Clebsch that couples the two doublets to $\mathbf{1}^{0-}$. The introduction of this new flavon is the only extension we make to the model in order to describe the neutrino phenomenology. After introducing ϕ_ν , the neutrino Dirac and Majorana mass matrices read

$$M_{LR} = \begin{pmatrix} 0 & l_1 \epsilon' & l_3 r_2 \epsilon' \\ -l_1 \epsilon' & l_2 \epsilon & l_3 r_1 \epsilon \\ 0 & l_4 \epsilon & 0 \end{pmatrix} \langle H_U \rangle, \quad M_{RR} = \begin{pmatrix} r_4 r_2^2 \epsilon'^2 & r_4 r_1 r_2 \epsilon \epsilon' & r_2 \epsilon' \\ r_4 r_1 r_2 \epsilon \epsilon' & r_3 \epsilon & r_1 \epsilon \\ r_2 \epsilon' & r_1 \epsilon & 0 \end{pmatrix} \Lambda_R, \quad (4.4)$$

where Λ_R is the right-handed neutrino mass scale, and we have parameterized the $O(1)$ coefficients. Furthermore, the charged lepton Yukawa matrix including $O(1)$ coefficients reads

$$Y_L \sim \begin{pmatrix} 0 & c_1 \epsilon' & 0 \\ -c_1 \epsilon' & 3c_2 \epsilon & c_3 \epsilon \\ 0 & c_4 \epsilon & 1 \end{pmatrix}. \quad (4.5)$$

The factor of 3 in the 22 entry is simply assumed at present, but originates from the Georgi-Jarlskog mechanism [15] in the grand unified case considered later.

The left-handed Majorana mass matrix M_{LL} follows from the seesaw mechanism

$$M_{LL} \approx M_{LR} M_{RR}^{-1} M_{LR}^T, \quad (4.6)$$

which yields

$$M_{LL} \sim \begin{pmatrix} (\epsilon'/\epsilon)^2 & \epsilon'/\epsilon & \epsilon'/\epsilon \\ \epsilon'/\epsilon & 1 & 1 \\ \epsilon'/\epsilon & 1 & 1 \end{pmatrix} \frac{\langle H_U \rangle^2 \epsilon}{\Lambda_R}, \quad (4.7)$$

where we have suppressed the $O(1)$ coefficients. We naturally obtain large mixing between second- and third-generation neutrinos, while the 12 and 13 mixing angles are $O(\epsilon'/\epsilon)$. However, taking into account the diagonalization of Y_L , the relative 12 mixing angle can be made smaller, as we discuss below. Explanation of the observed atmospheric neutrino fluxes by ν_μ - ν_τ mixing suggests $\sin^2 2\theta_{23} \gtrsim 0.8$ and $10^{-3} \lesssim \Delta m_{23}^2 \lesssim 10^{-2}$, while the solar neutrino deficit may be accommodated assuming the small-angle MSW solution $2 \times 10^{-3} \lesssim \sin^2 2\theta_{12} \lesssim 10^{-2}$ for $4 \times 10^{-6} \lesssim \Delta m_{12}^2 \lesssim 10^{-5}$, where all squared masses are given in eV^2 . We display

²Assuming more than one ϕ_ν leads to the same qualitative results.

below an explicit choice of the $O(1)$ parameters that yields both solutions simultaneously; a more systematic global fit will be presented in Ref. [10].

If M_{LL} and Y_L are diagonalized by $M_{LL} = VM_{LL}^0 V^\dagger$, $Y_L = U_L Y_L^0 U_R^\dagger$, then the neutrino CKM matrix is given by

$$V_\nu = U_L^\dagger V. \quad (4.8)$$

We aim to reproduce the 12 and 23 mixing angles, as well as the ratio $10^2 \lesssim \Delta m_{23}^2 / \Delta m_{12}^2 \lesssim 2.5 \times 10^3$ suggested by the data. Obtaining this ratio is sufficient since Λ_R is not determined by symmetry considerations and may be chosen freely. Assuming the previous values $\epsilon = 0.02$ and $\epsilon' = 0.004$ and the parameter set $(l_1, \dots, l_4, r_1, \dots, r_4, c_1, \dots, c_4) = (0.5, 1.0, -1.2, 2.3, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0)$, we find:

$$\frac{\Delta m_{23}^2}{\Delta m_{12}^2} = 105, \quad \sin^2 2\theta_{12} = 5 \times 10^{-3}, \quad \sin^2 2\theta_{23} = 0.9, \quad (4.9)$$

which fall in the desired ranges. While all our coefficients are of natural size, we have arranged for an $O(15\%)$ cancellation between 12 mixing angles in U_L and V to reduce the size of $\sin^2 2\theta_{12}$ to the desired value.

Case II: Here we assume that the flavons transform nontrivially under an $SU(5)$ GUT group, namely $A \sim \mathbf{1}$, $S \sim \mathbf{75}$, $\phi \sim \mathbf{1}$, and $\Sigma \sim \mathbf{24}$. Note that since $\overline{H} \sim \mathbf{5}$, the products $S\overline{H}$ and $A\overline{H}$ transform as a $\mathbf{45}$ and $\mathbf{5}$, respectively, ultimately providing a factor of 3 enhancement in the 22 entry of Y_L (the Georgi-Jarlskog mechanism). In addition, two $\mathbf{2}^{+0}$ doublets are introduced, $\phi_{\nu 1}$ and $\phi_{\nu 2}$, since the texture obtained for the neutrino masses by adding only one extra doublet is not viable. Both doublets ϕ_ν have vevs of the form displayed in Eq. (4.3). Crucially, the presence of these two new doublets does not alter the form of any charged fermion Yukawa texture.

The neutrino Dirac and Majorana mass matrices now take the form

$$M_{LR} = \begin{pmatrix} 0 & l_1 \epsilon' & l_5 r_2 \epsilon' \\ -l_1 \epsilon' & l_2 \epsilon^2 & l_3 r_1 \epsilon \\ 0 & l_4 \epsilon & 0 \end{pmatrix} \langle H_U \rangle, \quad M_{RR} = \begin{pmatrix} r_3 \epsilon'^2 & r_4 \epsilon \epsilon' & r_2 \epsilon' \\ r_4 \epsilon \epsilon' & r_5 \epsilon^2 & r_1 \epsilon \\ r_2 \epsilon' & r_1 \epsilon & 0 \end{pmatrix} \Lambda_R, \quad (4.10)$$

while the charged fermion mass matrix is the same as in Eq. (4.5). Using Eq. (4.6) one obtains the texture:

$$M_{LL} \sim \begin{pmatrix} (\epsilon'/\epsilon)^2 & \epsilon'/\epsilon & \epsilon'/\epsilon \\ \epsilon'/\epsilon & 1 & 1 \\ \epsilon'/\epsilon & 1 & 1 \end{pmatrix} \frac{\langle H_U \rangle^2}{\Lambda_R}. \quad (4.11)$$

If we now choose $(l_1, \dots, l_5, r_1, \dots, r_5, c_1, \dots, c_4) = (-1.0, 1.0, 1.0, 0.5, 1.0, 0.5, 1.0, 1.0, 1.0, -2.0, 1.0, 1.0, 1.0, 1.0)$, we find

$$\frac{\Delta m_{23}^2}{\Delta m_{12}^2} = 282, \quad \sin^2 2\theta_{12} = 6 \times 10^{-3}, \quad \sin^2 2\theta_{23} = 0.995. \quad (4.12)$$

Again these values fall in the desired ranges to explain the atmospheric and solar neutrino deficits, assuming an appropriate choice for Λ_R .

V. CONCLUSIONS

In this letter we have shown how to reproduce the quark and charged lepton Yukawa textures of the $U(2)$ model in their entirety, using a minimal non-Abelian discrete symmetry group. We showed that the representation structure of $T' \times Z_3$, in particular the existence of three distinct 2-dimensional irreducible representations, allows for solutions to the solar and atmospheric neutrino problems that require neither a modification of the simple charged fermion Yukawa textures of the $U(2)$ model nor the introduction of singlet neutrinos. The simplicity of the symmetry structure of our model suggests that a more comprehensive investigation of the space of possible models is justified. Work on alternative neutrino sectors as well as a more detailed phenomenological analysis of the models described here will be presented elsewhere [10].

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TABLES

Sample element	E	R	C_2, C_2R	C_3	C_3^2	C_3R	C_3^2R
Order of class	1	1	6	4	4	4	4
Order of element	1	2	4	6	3	3	6
$\mathbf{1}^0$	1	1	1	1	1	1	1
$\mathbf{1}^+$	1	1	1	η	η^2	η	η^2
$\mathbf{1}^-$	1	1	1	η^2	η	η^2	η
$\mathbf{2}^0$	2	-2	0	1	-1	-1	1
$\mathbf{2}^+$	2	-2	0	η	$-\eta^2$	$-\eta$	η^2
$\mathbf{2}^-$	2	-2	0	η^2	$-\eta$	$-\eta^2$	η
$\mathbf{3}$	3	3	-1	0	0	0	0

TABLE I. Character table of the double tetrahedral group T' . The phase η is $\exp(2\pi i/3)$.

FIGURES

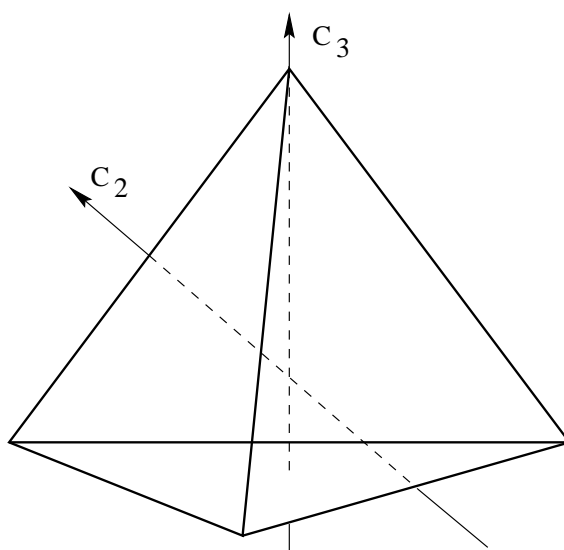


FIG. 1. Geometrical illustration of the group T' . The rotations C_2 and C_3 are defined in the text.